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MODELING THE VOLATITY OF CRYPTOCURRENCY MARKETS

The application of the model of geometric Brownian motion (GBM) for the problem of modeling and forecasting prices for cryptocurrencies is analyzed. For prediction the solution of the stochastic differential equation of the GBM model is used, which has a linear drift and diffusion coefficients. Different scenarios of price movement are considered. Fig.: 6. Refs.: 14 titles.

Keywords: geometric Brownian motion (GBM); modeling, forecasting; cryptocurrency.

Problem statement. At present time there is a problem of using information on market stock to save money and predict. There are many factors involved in price movements in the cryptocurrency market that are difficult to model: news, liquidity, market size, and even human psychology. Therefore, forecasting prices in a currency market is a hard work, but there are still valuable tools that can help us to understand price movements up to a certain point. For example, such tools are technical analysis, fundamental analysis, machine learning and also using formulas.

The arguments for using GBM [1] to predict cryptocurrency prices are:

1. The GBM process takes only positive values like real prices;
2. The GBM process is showing the same "roughness" on the way as real prices;
3. Calculations with GBM processes are relatively simple.

The disadvantages of using GBM [1] due to failed to correspond with reality are following:

1. Volatility of real prices changes over time on the currency market, but volatility is assumed to be constant (continuous) in GBM;
2. In real life prices often show jumps caused by various factors, but this path is continuous (no breaks) in GBM.

The purpose of the article is analysis the volatility of cryptocurrencies and improve the methods of this assessment.

Mathematical model. Brownian motion is a random motion of particles

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that never stops [2]. This random process has basic properties [1]:

1. Brownian motion is a Gaussian process i.e. the motion vectors caused by the process are normally distributed;
2. Brownian motion has stationary increments, i.e. its probability distribution does not change over time;
3. Brownian motion is a martingale, i.e. the best (in the sense of the mean square) prediction of the future movement of the process is its present state.

In addition to Robert Brown [2] this random process was considered by the following scientists: Jan Ingenhousz [3], Louis Bachelier (modeling price in financial markets) [4], Albert Einstein [5], Marian Smoluchowski [5], Paul Langevin [6] etc. The mathematical model of GBM has several approaches for the realization. The following are the main interpretations of this random process.

One of the approaches was implemented by A. Einstein, who discovered the formula for the diffusion coefficient of spherical Brownian particles [2]

$$D = \frac{RT}{6N_A \pi a \xi}, \tag{1}$$

where D is the diffusion coefficient, R is the universal gas constant, T is the absolute temperature, N_A is Avogadro's constant, a is radius of the particle, ξ is the dynamic viscosity.

The diffusion coefficient of a Brownian particle links the mean square of its shift x and the observation time t

$$\left\langle x^2 \right\rangle = 2Dt. \tag{2}$$

Einstein's main result is that the root-mean-square shift $\left\langle x^2(t) \right\rangle$ of a spherical Brownian particle grows linearly with time t with an amplitude inversely proportional to the number of molecules in the liquid [7], where $\left\langle x^2(t) \right\rangle = \left\langle \left| x(t) - x_0 \right|^2 \right\rangle = \frac{1}{N} \sum_{i=1}^{N} \left| x^{(i)}(t) - x^{(i)}(0) \right|^2$, N is the number of particles that need to be averaged; vector $x^{(i)}(0)$ is the initial position of the i^{th} particle and $x^{(i)}(t)$ is the position of the i^{th} particle at time t. Angle brackets mean taking the average value from the set of values of the expression bracketed.

The next approach was initiated by Paul Langevin. The Langevin equation [8] is an example of a stochastic differential equation and the random Langevin force is an illustration of a random process [9]. A stochastic

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differential equation is a differential equation containing a random process $\hat{\eta}(t)$, i.e. equation of the form

$$\frac{d\hat{x}(t)}{dt} = G(\hat{x}(t), t, \hat{\eta}(t)), \tag{3}$$

where G is a defined function that depends on the variable $x(t)$, on time t and on a random process $\hat{\eta}(t)$. A random process is a family $\hat{x}(t)$ of random variables depend on some continuous real parameter t. The function G can depend on the random process $\hat{\eta}(t)$ in an arbitrary way. However, it can be described by stochastic differential equations in which $\hat{\eta}(t)$ is linear, that is

$$\frac{d\hat{x}(t)}{dt} = q\left(\hat{x}(t)\right) + g\left(\hat{x}(t)\right)\hat{\eta}(t). \tag{4}$$

Stochastic differential equations of this type are called Langevin equations. In this case an independent random process $\hat{\eta}(t)$ is usually called noise.

Following this notation, the term $g\left(\hat{x}(t)\right)\hat{\eta}(t)$ in equation (4) is called the noise or diffusion term, whereas $q\left(\hat{x}(t)\right)$ is the deterministic term or drift term. There is a case when the function $g\left(\hat{x}(t)\right)$ is constant and in this case the noise is called additive. Otherwise the noise is called multiplicative. For simplicity of notation the Langevin differential equation (4) will be written in the form

$$\frac{dx(t)}{dt} = q(x(t)) + g(x(t))\eta(t). \tag{5}$$

Modification of the algorithm in the GBM model. To modify the algorithm in the model of GBM, the motion $B(t)$ is described in the form of a stochastic solution (6), which has a linear drift and diffusion coefficients

$$\frac{dB(t)}{B(t)} = \mu dt + \sigma dW(t). \tag{6}$$

If the initial value of the Brownian motion is equal to $B(t) = B_0$ and the calculation of $\sigma B(t)dW(t)$ can be applied with Ito's lemma (the formula of

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change of variable in a stochastic differential equation, $F(x) = \log(X)$ [10]) then we get equation (7)

$$B_k = e^{\log B_0 + drift_k t_k + diffusion_k} = B_0 e^{drift_k t_k + diffusion_k}, \; k = \overline{1, M}, \tag{7}$$

here

$$drift_k = \mu - \frac{1}{2}\sigma^2, \tag{8}$$

$$diffusion_k = \sigma W_k. \tag{9}$$

Thus, we have the equation (10)

$$B_k = B_0 e^{(\mu - \frac{1}{2}\sigma^2)t_k + \sigma W_k}, \; k = \overline{1, M}. \tag{10}$$

Geometric Brownian motion is widely used to model prices in the market [11, 12]. The following is an algorithm for modeling prices in the financial market using GBM [13]:

1. Determination of the initial price B_0 – the last price of the historical prices.

2. Determination of time step dt – increment time step (For example day, week, month etc.).

3. Determining the length of time T for forecasting (i.e. how many time points are needed to forecast prices).

4. Determination of the parameter of time points N that is equal to $\frac{T}{dt}$.

5. Building an array t for time points (For example an array of t starts from 1 to 30 for a month).

6. Finding the average return μ of historical prices that is equal to $\frac{1}{|k|}\sum_{k=1}^{M}\frac{B_k - B_{k-1}}{B_{k-1}}$.

7. Finding the standard deviation σ that equals.

Building an array $b = [z_1, ..., z_k]$ of Brownian increment. In order to have different scenarios of price movement the parameter *size_scenarios* needs that equals n, $n = \overline{1, \infty}$. Array b stores a random number for each appropriate prediction time point that is obtained from the standard normal distribution. These random numbers will add random shocks to the model.

8. Then the *numpy.random.normal*() function is used to generate random values.

9. Building an array W of the Brownian path that determines how prices move from the starting point B_0 to some other point in time t. This parameter is equal to $\sum_{i=1}^{k} b_i$.

10. Determination of the drift that equals $\mu - \frac{1}{2}\sigma^2$ and diffusion that equals $\sigma b_k = \sigma z_k$ for each point k.

11. Determination of the geometric Brownian motion for each next point (price) B_k using the formula $B_k = B_0 e^{t_k\left(\mu - \frac{1}{2}\sigma^2\right)} + \sigma W_k$.

The results of the work of the algorithm. This algorithm is done in the Python programming language and the 2021 BTC / USD prices from Yahoo Finance [14] was used for result testing.

The x-axis of the graph (Fig. 1) shows the days from January 1, 2021 to April 30, 2021, and the y-axis of the graph (Fig. 1) shows prices in dollars. The general trend is upward. Random shocks of falling prices occur daily that resulting in an irregular chart curve.

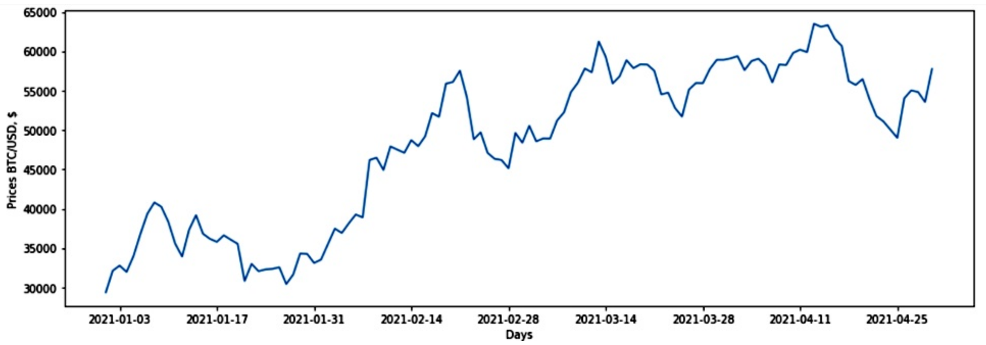

Fig. 1. BTC / USD prices for January – April 2021

The period for historical prices is one month – March, and for the price forecast – April.

The chart (Fig. 2) shows two scenarios for forecasting prices. One scenario (blue line) goes up and the second line ranges at relatively the same level. At the beginning prices decline slightly, then gradually rise. It can be observed from the drift array that the values are positive, i.e. the long-term price trend is upward. If the drift is negative, then the forecast may rise and this is due to the random shocks that are created using the standard normal random value. The black dotted line is the real prices for April. The correlation

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coefficients are shown for two forecasts as to the real prices on the chart (Fig. 2). The yellow forecast line shows greater relationship $r = -0.7547$ than the blue line illustrates.

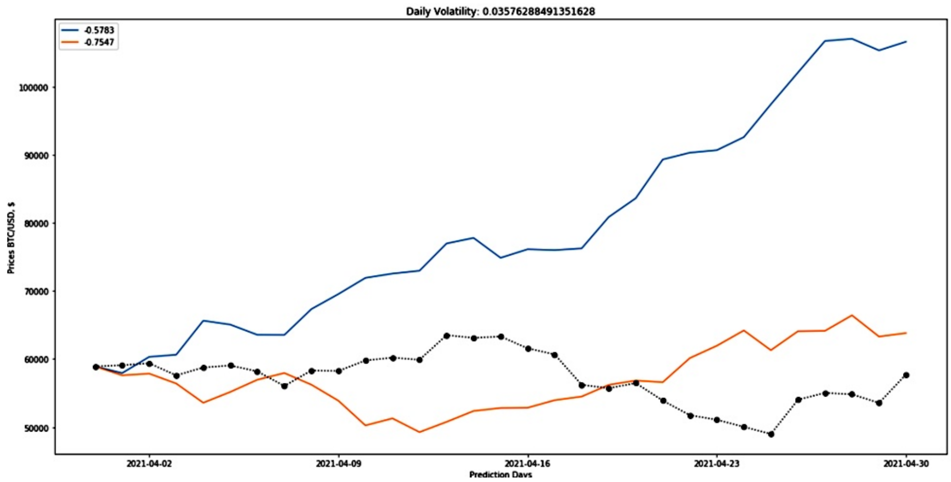

Fig. 2. Two price scenarios for April (1 month for historical prices, 1 month for forecast prices)

If parameter size_scenarios equals 25 then the chart (Fig. 3) has been obtained. Each run has a different price scenario. Every time the model is run, there will be a different W array and this will result in the different predictions.

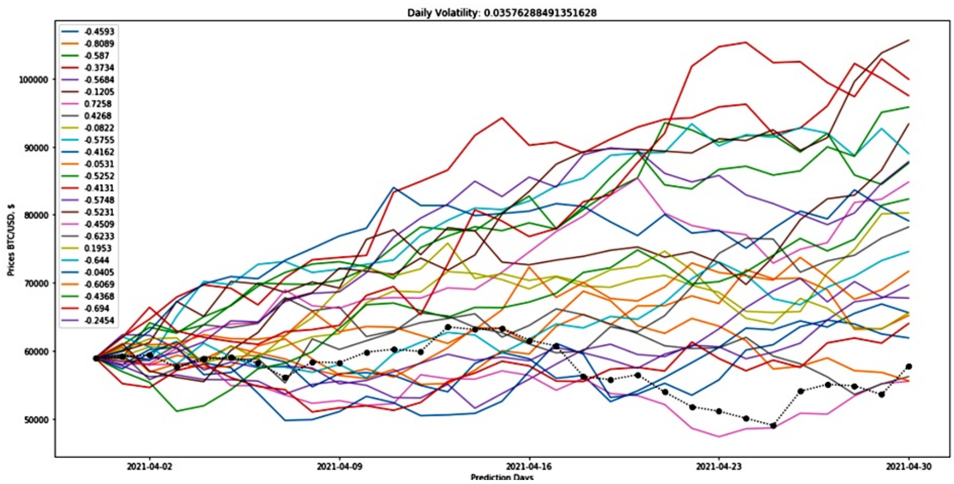

Fig. 3. 25 different price scenarios for April

If the historical price period contains 3 months (January, February and March) and April is also used to forecast, then the chart (Fig. 4) has been obtained. The results changed slightly.

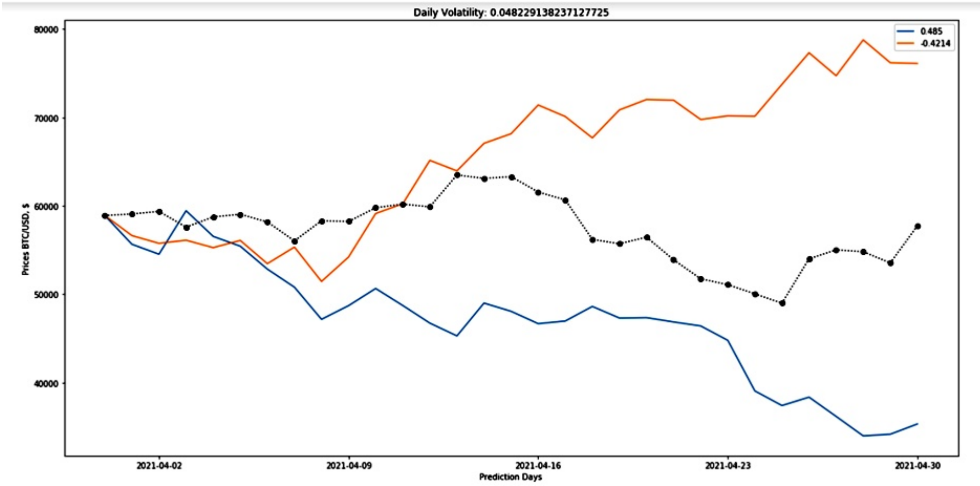

Fig. 4. Two scenarios of prices for April (3 months for historical prices and 1 month for forecast)

If the parameter of the length of time T = 22 for the forecast, then the chart (Fig. 5) has been obtained, on which the results are similar.

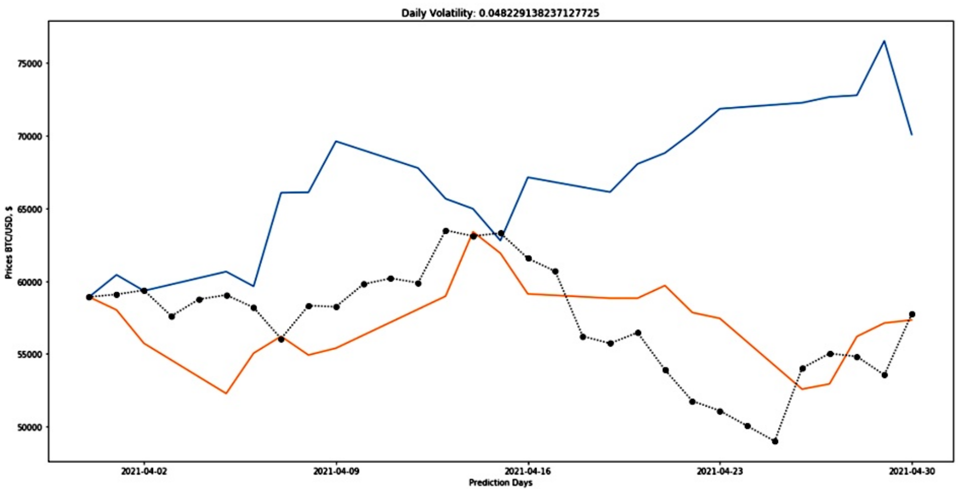

Fig. 5. Two scenarios of prices for April (3 months for historical prices, 1 month for forecast prices, T = 22)

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If the forecasting period contains 5 months from April to August, then the chart (Fig. 6) has been obtained:

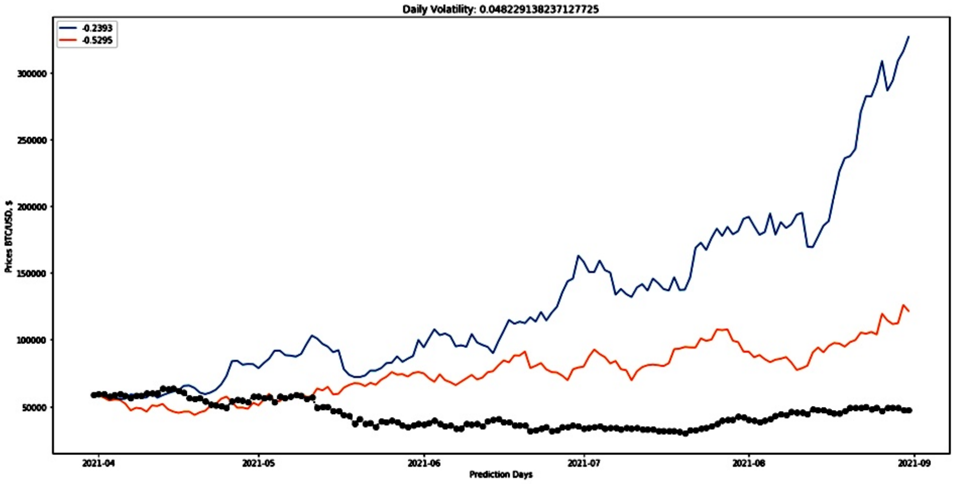

Fig. 6. Two scenarios of prices from April to August (3 months for historical prices, 1 month for forecast prices, $T = 153$)

The graph (Fig. 6) shows that the longer the period for the forecast, the more the forecast prices move away from the real prices. The second scenario (yellow line) predicts the real price trend better than the first scenario (blue line).

Conclusion. This work enables to conclude that the method of building a geometric Brownian motion allows to model the prices of cryptocurrencies, make forecasts and see the trend in pricing.

The test results of this algorithm showed that the longer the forecast period is given to the program input, the more the forecast prices move away from the real prices. Also, as can be noted that if to increase the value of the historical price parameter, then the price forecast result will not be better, as, for example, in machine learning. Parameters μ and σ remain similar in value.

In this work the main mathematical methods for finding Brownian motion as a random process have been considered, for example, Einstein's method, Langevin's method using a stochastic differential equation. It also shows how a modification of the GBM algorithm helps to see which paths prices might follow. This gives us the ability to create robust strategies of trading and hedging that we can rely on or be focused by.

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Досліджується застосування моделі геометричного броунівського руху для задач моделювання та прогнозування цін на криптовалюти. Для прогнозування використовується рішення стохастичного диференціального рівняння моделі GBM, яке має лінійний дрейф та коефіцієнти дифузії. В алгоритмі розглянуті різні сценарії поведінки цін. Іл.: 6. Бібліогр.: 14 назв.

Ключові слова: модель геометричного броунівського руху (GBM); моделювання; прогнозування; криптовалюта.

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Исследуется применение модели геометрического броуновского движения для задачи моделирования и прогнозирования цен на криптовалюты. Для прогнозирования используется решение стохастического дифференциального уравнения модели GBM, которое имеет линейный дрейф и коэффициенты диффузии. В алгоритме рассмотрены разные сценарии поведения цен. Іл.: 6. Бібліогр.: 14 назв.

Ключевые слова: модель геометрического броуновского движения (GBM); моделирование; прогнозирование; криптовалюта.

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The application of the model of geometric Brownian motion for the problem of modeling and forecasting prices for cryptocurrencies is analyzed. For prediction the solution of the stochastic differential equation of the GBM model is used, which has a linear drift and diffusion coefficients. Different scenarios of price movement are considered. Fig.: 6, Refs.: 14 titles.

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