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# THE PROBLEMS OF RATIONAL ALLOCATION OF A MULTIDIMENSIONAL RESOURCE FOR MULTI-NOMENCLATURE PRODUCTION IN CONDITIONS OF UNCERTAINTY OF THE INITIAL DATA

Object of research: the problem of rational allocation of a multidimensional resource to ensure multi-nomenclature production in conditions of uncertainty. Mathematical model of the problem: nonlinear non-separable multi-index mathematical programming problem based on a system of linear multi-index constraints. An important feature complicating the problem is the fuzziness of the source data. Convergent iterative procedure based on the proven theorem is proposed to solve the problem. The computational procedure consist of two stages. At the first stage, an acceptable, constraint-satisfying initial solution to the problem is found, which is improved step by step at the second stage.. Іл.: 3. Бібл. 11 назв..

**Keywords:** rational distribution of a multidimensional resource, multi-nomenclature production, nonlinear optimization, fuzzy source data.

**Introduction.** Numerous planning problems in engineering, economics, military affairs, etc. are reduced to a mathematical model characteristic of the so-called problems of rational allocation of a limited resource [1 – 4]. This model has the following form: a linear or nonlinear, but separable objective function of many variables, the sum of which is subject to linear constraints. Along with this, in many practical applications, more complex models are proposed to describe the function that determines the income received from resource allocation. These models set nonlinear problems of distributing a multidimensional resource across the elements of a multi-nomenclature production using Cobb-Douglas type functions [4]. The canonical problem of the rational distribution of a multidimensional resource in the production of a diversified product is formulated as follows: to find a rational plan for the distribution of a limited multidimensional resource that ensures maximum profit from its implementation. The known results of solving such a problem

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correspond to a simple situation when the distributed resource is one-dimensional [6]. At the same time, the following are introduced:

$x = (x_1, x_2, ..., x_n)$ – a vector specifying the desired resource allocation,

$C_0$ – total distributed resource,

$\varphi_j(x_j)$ – a function that determines the profit from the sale of the jth product of production, $j = 1, 2, \ldots, n$.

The mathematical model of the problem looks like this: find a set that $X = (x_1, x_2, ..., x_n)$ – maximizes the total profit

$$\phi(x)=\sum_{j=1}^{n} \varphi_j(\mathrm{x}_j) \tag{1}$$

and satisfying the constraints

$$\sum_{j=1}^{n} \mathrm{x_j} = C_0, \; x_j \geq 0, \; j = 1, 2, \ldots, n. \tag{2}$$

Let's say, for example, $\varphi_j(x_j)=a_{0j}x_j^{a_{1j}}$. Then the total income from the implementation of the plan will be equal to $\phi(x)=\sum_{j=1}^{n} a_{0j}x_j^{a_{1j}}$.

The problem is solved by the method of indeterminate Lagrange multipliers. Let's introduce the Lagrange function

$$\tilde{\phi}(x)=\sum_{j=1}^{n} a_{0j}x_j^{a_{1j}} -\lambda\left(\Sigma_{j=1}^{n}x_j-C_0\right).$$

Further

$$\frac{d\tilde{\phi}(x)}{dx_j}=a_{0j}a_{1j}x_j^{a_{1j}-1}-\lambda=0, \; j = 1, 2, \ldots, n.$$

From here

$$x_j=\left(\frac{\lambda}{a_{0j}a_{1j}}\right)^{\frac{1}{a_{1j}-1}}, \; j = 1, 2, \ldots, n. \tag{3}$$

Substituting (3) into (2), we obtain an equation with respect to an unknown parameter λ

$$\sum_{j=1}^{n} \left(\frac{\lambda}{a_{0j}a_{1j}}\right)^{\frac{1}{a_{1j}-1}} = C_0. \tag{4}$$

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Equation (4) is solved numerically, however, an analytical solution can be easily obtained if $a_{1j} = a_1, j = 1, 2, \ldots, n$. In this case, the equation is simplified to the form:

$$\left(\frac{\lambda}{a_1}\right)^{\frac{1}{a_1-1}} \sum_{j=1}^{n} \left(\frac{1}{a_{0j}}\right)^{\frac{1}{a_1-1}} = C_0,$$

from where

$$\left(\frac{\lambda}{a_1}\right)^{\frac{1}{a_1-1}} = \frac{C_0}{\sum_{j=1}^{n}\left(\frac{1}{a_{0j}}\right)^{\frac{1}{a_1-1}}} \quad (5)$$

Now, substituting (5) into (3), we get the desired solution to the problem:

$$x_j = \left(\frac{\lambda}{a_1}\right)^{\frac{1}{a_1-1}} \left(\frac{1}{a_{0j}}\right)^{\frac{1}{a_1-1}} = \frac{C_0\left(\frac{1}{a_{0j}}\right)^{\frac{1}{a_1-1}}}{\sum_{j=1}^{n}\left(\frac{1}{a_{0j}}\right)^{\frac{1}{a_1-1}}}, j = 1, 2, \ldots, n. \quad (6)$$

The above technology for solving a one-dimensional problem cannot be directly extended to a multidimensional case. In this regard, we formulate the purpose of the study: to develop a method for solving the problem of rational distribution of a multidimensional resource by elements of a multi-nomenclature production.

Main result. Let's introduce a matrix $X = (x_{ij}), x_{ij}$ is the amount of a resource of $i$-th type planned for the manufacture of $j$-th product, a vector $A = (a_1, a_2, \ldots, a_m)$ is determining the distribution of a given number of units of a multidimensional resource, and a vector $C = (c_1, c_2, \ldots, c_m)$ of unit costs of the corresponding resources. Let's further assume $\varphi_j(x_j) = \prod_{i=1}^{m} x_{ij}^{\beta_{ij}}$ is a function determining the profit received from the sale of $j$-th product during distribution of resource $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$.

Now we formulate the following problem: to find a matrix $X(x_j)$, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n,$ maximizing the total profit corresponding to the distribution of a multidimensional resource $(a_1, a_2, \ldots, a_m)$, determined by the ratio

$$F(X) = \sum_{j=1}^{n} \prod_{i=1}^{m} x_{ij}^{\beta_{ij}} \quad (7)$$

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and satisfying the constraints

$$\sum_{j=1}^{n} x_{ij}=a_i,\ i = 1, 2, \dots, m, \tag{8}$$

$$\sum_{i=1}^{m} c_i x_{ij}=b_j, j = 1, 2, \dots, n, \tag{9}$$

$$\sum_{i=1}^{m} a_i=\sum_{j=1}^{n} b_j. \tag{10}$$

Here $b_j$ is the cost consumed in the manufacture of the $j$ product corresponding to the plan $X$.

The resulting model (7) – (10) sets a rather complex mathematical programming problem with a nonlinear non-separable objective function and constraints typical for distributive linear programming problems. It is difficult to directly solve such a problem using standard mathematical programming methods. The only real approach that can be used is implemented by the zero-order method for the case when $m$ and $n$ are small. However, an approximate solution can be obtained as follows.

We transform the original problem by reducing its constraints to a form typical for two-index linear programming transport problems. To do this, we will introduce new variables $Z_{ij} = C_i X_{ij}$, $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. Substituting these variables into constraints (8) and (9), we approximately obtain

$$\sum_{j=1}^{n} Z_{ij}=c_i a_i=d_i,\ i = 1, 2, \dots, m, \tag{11}$$

$$\sum_{i=1}^{m} Z_{ij} = b_j,\ j = 1, 2, \dots, n, \tag{12}$$

$$z_{ij} \geq 0,\ i = 1, 2, \dots, m,\ j = 1, 2, \dots, n. \tag{13}$$

Here

$Z_{ij}$ is the cost of $i$-th resource consumed in the manufacture of $x_{ij}$ units of $j$-th product. At the same time, we transform the target function (7):

$$F(Z)=\sum_{j=1}^{n} \prod_{i=1}^{m} \left(\frac{Z_{ij}}{C_i}\right)^{\beta_{ij}} =\sum_{j=1}^{n} G_j \prod_{i=1}^{n} Z_{ij}{}^{\beta_{ij}}=$$

$$= \sum_{j=1}^{n} G_j \prod_{i=1}^{m} \varphi_{ij}(Z_{ij}), \tag{14}$$

where

$$G_j=\prod_{i=1}^{m} \left(\frac{1}{C_i}\right)^{\beta_{ij}},\ j = 1, 2, \dots, n.$$

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The solution of the problem is obtained using a two-step procedure based on the following theorem.

Theorem. For a set $\{Z_{ij}{}^*\}$ to be a solution to problem (11) – (14) it is necessary and sufficient that this set, satisfying (11) – (13), maximises the

$$\phi_j(Z_{ij}) = \prod_{i=1}^{m} \phi_j(Z_{ij}),$$

that is, it took place

$$\prod_{i=1}^{m} \varphi_{ij}(z_{ij}^*) = \max_{\{z_{ij}\}} \left\{\prod_{i=1}^{m} \varphi_{ij}(z_{ij})\right\} \quad (15)$$

for all $j = 1, 2, \ldots, n$.

Sufficiency. Let $\{Z_{ij}^{(0)}\}$ be an arbitrary set satisfying (11) – (13) and not coinciding with $\{Z_{ij}{}^*\}$. By virtue of (15) we have

$$\prod_{i=1}^{m} \varphi_{ij}(z_{ij}^*) \geq \prod_{i=1}^{m} \varphi_{ij}(z_{ij}^{(0)}),\ j = 1, 2, \ldots, n.$$

Then

$$\sum_{j=1}^{n} \prod_{i=1}^{m} \varphi_{ij}(z_{ij}^*) \geq \sum_{j=1}^{n} \prod_{i=1}^{m} \varphi_{\mathrm{ij}}(z_{ij}^{(0)}). \quad (16)$$

The resulting inequality means that when the requirements of the theorem are met, the set $\{Z_{ij}{}^*\}$ is the problem solution.

Necessity. We show that the optimality of the set $\{Z_{ij}{}^*\}$ as a whole implies its column optimality.

Let's assume the opposite, that there is a set $\{Z_{ij}{}^{**}\}$ , that is optimal in general and does not coincide with the column-optimal set $\{Z_{ij}{}^*\}$. It follows from optimality $\{Z_{ij}{}^{**}\}$ in general that

$$\sum_{i=1}^{n} \prod_{i=1}^{m} \varphi_{ij}(z_{ij}^{**}) \geq \sum_{i=1}^{n} \prod_{i=1}^{m} \varphi_{j}(z_{ij}),$$

for any set $\{Z_{ij}\}$, including for a set $\{Z_{ij}{}^*\}$ that does not match with $\{Z_{ij}{}^{**}\}$, that is

$$\sum_{i=1}^{n} \prod_{i=1}^{m} \varphi_{ij}(z_{ij}^{**}) \geq \sum_{j=1}^{n} \prod_{i=1}^{m} \varphi_{ij}(z_{ij}^*). \quad (17)$$

On the other hand, from column optimality $\{Z_{ij}{}^*\}$ it follows that

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$$\prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}^{*}\right) \geq \prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}\right),$$

for any set $\{Z_{ij}\}$, including for a set $\{Z_{ij}^{**}\}$ that does not match with $\{Z_{ij}^{*}\}$, that is

$$\prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}^{*}\right) \geq \prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}^{**}\right),$$

from where

$$\sum_{j=1}^{n} \prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}^{*}\right) \geq \sum_{j=1}^{n} \prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}^{**}\right). \quad (18)$$

The resulting inequalities (17) and (18) contradict each other. Thus, the assumption about the possibility of the existence of a set that is optimal in general, but suboptimal in columns has led to a contradiction. The theorem has been proved.

Using this theorem, the solution of the initial problem at the first stage is reduced to the sequential solution of nproblems (for $j = 1, 2, \ldots, n$) of the type: find a set $\{Z_{ij}\}$ that maximizes

$$\phi_j\left(Z_{ij}\right) = \prod_{i=1}^{m} \varphi_{ij}\left(z_{ij}\right),$$

and satisfying the constraints

$$\sum_{i=1}^{m} z_{ij} = b_j, \ j = 1, 2, \ldots, n$$

If, as a result of solving all nproblems, a matrix $(Z_{ij})$ is obtained, whose components satisfy the constraints (11), then this matrix, after returning to the original variables $x_{ij}$, determines the desired solution to the problem. Otherwise, at the second stage, a step-by-step correction of this matrix is performed until a solution satisfying all constraints is obtained. Let's consider the proposed procedure in more detail.

The first stage. For each $j$, $j = 1, 2, \ldots, n$, the problem of finding a vector $Z_j=(Z_{ij}, Z_{zj}, \ldots, Z_{mj})$ maximizing

$$\varphi_j\left(z_j\right) = \prod_{i=1}^{m} z_{ij}^{\beta_{ij}} \quad (19)$$

and satisfying is solved sequentially (12). In this case, the pre-multiplicative criterion (19) is converted into an additive one by logarithm. We have

$$L_j\left(z_j\right) = l_n \, \phi_j\left(z_j\right) = \sum_{i=1}^{m} \beta_{ij} \, l_n \frac{z_{ij}}{C_i}.$$

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The solution is obtained using the method of indefinite Lagrange multipliers. Let's introduce the Lagrange function:

$$\phi_j(z_j)=\sum_{i=1}^{m}\beta_{ij}\,l_n\frac{z_{ij}}{C_i}-\lambda_j\left(\sum_{i=1}^{m}z_{ij}-b_j\right).$$

Further

$$\frac{d\phi_j(Z_j)}{dZ_{ij}}=\beta_{ij}\frac{C_i}{z_{ij}}-\lambda_j=0,$$

from where

$$Z_{ij}=\frac{\beta_{ij}C_i}{\lambda_j},\ i=1,2,\ldots,m. \qquad (20)$$

Substituting (20) into (12), we obtain the equation with respect to $\lambda_j$:

$$\sum_{i=1}^{m}Z_{ij}=\sum_{i=1}^{m}\frac{\beta_{ij}C_i}{\lambda_j}=\frac{1}{\lambda_j}\sum_{i=1}^{m}\beta_{ij}C_i=b_j\ ,$$

from where

$$\frac{1}{\lambda_j}=\frac{b_j}{\sum_{i=1}^{m}\beta_{ij}C_i}$$

Substituting this result in (20), we get

$$Z_{ij}^{(0)}=\frac{\beta_{ij}b_j\cdot C_i}{\sum_{i=1}^{m}\beta_{ij}\cdot C_i},\ i=1,2,\ldots,m, j=1,2,\ldots,n. \qquad (21)$$

The matrix ($Z_{ij}^{(0)}$), obtained as a result of the sequential solution of this problem for all $j$, $j = 1, 2, \ldots, n$, determines its initial solution. This solution is checked to satisfy its limitations (11). For this purpose, the calculation is performed:

$$\varepsilon_i=\sum_{i=1}^{m}Z_{ij}^{(0)}-d_i,\ i=1,2,\ldots,m. \qquad (22)$$

If, at the same time, it turns out for all $i \in (1; 2; \ldots; m)$ that $\varepsilon_i \le 0$, then the resulting matrix $Z^{(0)}$ satisfies all the constraints, in accordance with this, it is a solution to the problems. Otherwise, the transition to the second stage is carried out.

At the second stage, the correction of the matrix obtained at the first stage is carried out. In this case, two subsets are distinguished from the set $E=\{1; 2; \ldots; m\}$ of rows of the matrix $Z^{(0)}$:

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$$E^{(+)}=\{i \in E,\ \sum_{i=1}^{m} Z_{ij}^{(0)}\text{-}d_i > 0\},$$

$$E^{(-)}=\{i \in E,\ \sum_{i=1}^{m} Z_{ij}^{(0)}\text{-}d_i < 0\},$$

The lines belonging to $E^{(+)}$ are called redundant, and the lines belonging to $E^{(-)}$ are called insufficient. Next, the matrix $Z^{(0)}$ is viewed by columns. In the next column$j$, two subsets $E_j^{(+)}$ and $E_j^{(-)}$ are highlighted . The first subset includes elements $Z_{ij} > 0$ lying in redundant rows, that is $E_j^{(+)}=\{Z_{ij} > 0,\ i \in E^{(+)}\}$. The second subset includes elements $Z_{ij} > 0$ that lie in insufficient rows, that is $E_j^{(-)}=\{Z_{ij} > 0,\ i \in E^{(-)}\}$. It is clear that if in this $j$column, from any element in this column, for example $Z_{i_0 j}$, belonging to a subset $E_j^{(+)}$, subtract some specially selected number Δ, and then add this number to any element of the same column, for example, $Z_{k_0 j}$ belonging to a subset $E_j^{(-)}$, then the row with the number will $i_0$ become less redundant, and the row with a number $k_0$ is less than insufficient. Note that such a change in the plan will not lead to a violation of the restrictions (12). On the other hand, it is clear that the new plan resulting from the correction will be worse than the previous plan $Z^{(0)}$. Therefore, from the set of possible pairs $(Z_{i_0 j}, Z_{k_0 j})$, it is advisable to choose the one for which this deterioration will be the least. Thus, it is necessary to establish the rules for choosing the correction pair and the values of the correction parameter Δ. These rules are simple and naturally follow from the above-formed meaning and technology of the described correction procedure:

$$(j_0, i_0, k_0)=$$

$$\arg\min[G_{j_0}[\begin{matrix} \left(Z_{i_0 j_0}^{(0)}\right)^{\beta_{i_0 j_0}} \left(Z_{k_0 j_0}^{(0)}\right)^{\beta_{k_0 j_0}} - \\ -\left(Z_{i_0 j_0}^{(0)}-\Delta\right)^{\beta_{i_0 j_0}} \left(Z_{k_0 j_0}^{(0)}+\Delta\right)^{\beta_{k_0 j_0}} \end{matrix}]] \qquad (23)$$

$$\Delta = \min_{\substack{i_0 \in E^{(+)} \\ k_0 \in E^{(-)}}} \{Z_{i_0 j_0}^{(0)} - \varepsilon_{k_0}\} \qquad (24)$$

The choice of coordinates of the corrected elements according to rule (23) ensures the minimum possible decrease in the objective function (14) as a result of the adjustment.

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The choice of a value Δ in accordance with (24) is imposed by restriction (13) on the one hand, and on the other by the inexpediency of transferring an insufficient string to $k_o$ a group of redundant strings.

As a result of the adjustment step according to rules (23), (24), a new plan will be received:

$$Z_{ij}^{(1)} = \{ \begin{array}{l} Z_{ij}^{(0)} - \Delta,\ i = i_o, j_o = j_o \\ Z_{ij}^{(0)} + \Delta,\ i = k_o, j = j_o \\ Z_{ij}^{(0)}, (j \neq j_o) \cup [(j = j_o) \cap (i \neq i_o) \cap (i \neq k_o)] \end{array}$$

The described correction procedure continues until all constraints (11) are fulfilled, which means the end of the problem solution.

The considered problem becomes significantly more difficult if its initial data cannot be estimated accurately and, therefore, described, for example, in terms of fuzzy mathematics [5, 6]. A traditionally unused approximate method for solving such a problem is as follows [7]. Suppose that the parameters $C_1, C_2, \ldots, C_m$ of the problem (7) – (11) are fuzzy numbers of $(L - R)$-type and are given by the corresponding membership functions, that is $M_i(C_i) = <m_i, \alpha_i, \beta_i>$. To solve the initial problem in this case, we use the above-described method of finding a rational distribution of a multidimensional resource. In this case, we will set the numerical values of the fuzzy parameters $C_1, C_2, \ldots, C_m$ equal to their modal values $m_1, m_2, \ldots, m_m$. Then the constraints (11), the objective function (7), and the result obtained after the first stage of the optimization procedure (21) will take the form:

$$\sum_{j=1}^{n} Z_{ij} = \sum_{j=1}^{n} m_i C_i = \alpha_i \qquad (25)$$

$$F(Z) = \sum_{j=1}^{n} \prod_{i=1}^{m} \left(\frac{Z_{ij}}{m_i}\right)^{\beta_{ij}} \qquad (26)$$

$$Z_{ij}^{(0)} = \frac{\beta_{ij} \cdot b_j m_i}{\sum_{i=1}^{m} \beta_{ij} m_i},\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \qquad (27)$$

The obtained result (27) can be used in solving practical problems if the value $\alpha_i$, $\beta_i$ is small compared to $m_i$. Otherwise, not taking into account the level of blurriness of the membership functions of fuzzy parameters, the problem can lead to gross errors.

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Fundamentally, another approach to solving the problem of rational allocation of a multidimensional resource in conditions of uncertainty is more adequate. The technology of its implementation is two-stage. At the first stage, a fuzzy problem is solved using some clear model of this problem, for example, assuming that all fuzzy parameters of the problem are replaced by their modal values. The resulting analytical expression for the optimized objective function and the corresponding set of variable variables is the basis for the second stage of the procedure. At this stage, using any zero-order optimization method (for example, Nelder-Meade), an iterative improvement of the obtained initial set of variables obtained after the first stage is carried out. In this case, a complex criterion is used, determined by the ratio

$$\phi(X_1,C)=F(X_1,C)\prod_{i=1}^{m}\mathrm{C_i}\left(X_{ij}\right).$$

Here, for the considered problem of rational resource allocation

$$F(x)=\sum_{j=1}^{n}\prod_{i=1}^{m}\left(X_{ij}\right)^{\beta_{ij}}C_i(x_{ij}).$$

At the same time, the set of variables obtained at the next iteration of the improvement that satisfies all the constraints is adjusted by the method described above.

**Conclusions.**

1)The problem of rational distribution of a multidimensional resource in the production of a diversified product is considered. A method is proposed for obtaining a solution to the nonlinear non-separable optimization problem arising in this case on a system of linear constraints for a set of two-index variables. To implement the method, a convergent iterative procedure based on the proven theorem is introduced. A method for solving the problem of allocating a limited resource is considered for the case when the parameters of the problem are not clearly defined. An iterative procedure is proposed to provide the desired solution.

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**Проблеми раціонального розподілу багатовимірного ресурсу для багатономенклатурного виробництва в умовах невизначеності вихідних даних / Раскін Л., Сухомлин Л., Гатунов А., Соколов Д.** // Вісник НТУ "ХПІ". Серія: Інформатика та моделювання. – Харків: НТУ "ХПІ". – 2025. – № 1 (13). – С. 22 – 34.

Об'єкт дослідження: задача раціонального розподілу багатовимірного ресурсу для забезпечення багатономенклатурного виробництва в умовах невизначеності. Математична модель задачі: нелінійна нероздільна багатоіндексна задача математичного програмування, що базується на системі лінійних багатоіндексних обмежень. Важливою особливістю, що ускладнює задачу, є нечіткість вихідних даних. Для розв'язання задачі пропонується конвергентна ітераційна процедура, що базується на доведеній теоремі. Обчислювальна процедура складається з двох етапів. На першому етапі знаходиться прийнятний, що задовольняє обмеження, початковий розв'язок задачі, який крок за кроком удосконалюється на другому етапі. Refs.: 11 titles.

**Ключові слова:** раціональний розподіл багатовимірного ресурсу, багатономенклатурне виробництво, нелінійна оптимізація, нечіткі вихідні дані.

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**The problems of rational allocation of a multidimensional resource for multi-nomenclature production in conditions of uncertainty of the initial data / Raskin L., Sukhomlyn L., Hatunov A., Sokolov D.** // Herald of the National Technical University "KhPI". Series of "Informatics and Modeling". – Kharkov: NTU "KhPI". – 2025. – № 1 (13). – P. 22 – 34.

Object of research: the problem of rational allocation of a multidimensional resource to ensure multi-nomenclature production in conditions of uncertainty. Mathematical model of the problem: nonlinear non-separable multi-index mathematical programming problem based on a system of linear multi-index constraints. An important feature complicating the problem is the fuzziness of the source data. Convergent iterative procedure based on the proven theorem is proposed to solve the problem. The computational procedure consist of two stages. At the first stage, an acceptable, constraint-satisfying initial solution to the problem is found, which is improved step by step at the second stage. Refs.: 11 titles.

**Keywords:** rational distribution of a multidimensional resource, multi-nomenclature production, nonlinear optimization, fuzzy source data.