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I. P. ZANEVSKYY, dr. tech. sci., prof, Lviv State University of Physical Culture named after Ivan Boberskyj, Lviv

VIBRATIONS IN THE OPEN COMPOUND KINEMATICAL CHAIN

The aim of the research was to create a model of vibration processes in the compound open kinematical chain with an external link. A mechanical and mathematical model of the lateral oscillations of the system during the external link accelerated motion is proposed. Correlation between longitudinal acceleration and natural frequencies are obtained. There are recommendations regarding determinations of the virtual forms of external link's vibration. The models and methods have been adapted for realization in the engineering method using well-known mathematical CAD systems. An example of the mechanical-mathematical model of a complex open kinematic chain allows us to study complex dynamic processes of the impulse nature that characterize the behavior of the "arrow-bow" system, in particular, vibrations and loss of stability of the arrow during its joint movement with the bow. The proposed approach to simulation of vibration processes in a complex open kinematic chain allows carrying out the implementation by numerical methods from a set of standard subroutines of widely available SCM packages. Il.: 3. Ref.: 7 Items

Keywords: model, vibration, compound, kinematic chain, Eigenform.

Introduction. A simple open kinematic chain is widely used to simulate dynamic processes in manipulators of industrial robots [1]. The design of a model of the bow-arrow system is based on a complex open kinematic chain [2]. The boom is modeled by a rod that forms only one kinematic pair – a hinge, which makes the kinematic chain open. This hinge (bowstring socket) connects three kinematic links – two branches of the bowstring and an arrow, which makes the kinematic chain complex.

The joint movement of the arrow with the bow is characterized by intense dynamic processes, in particular, dynamic longitudinal bending and vibrations. The lateral (in the projection on the transverse plane of the bow) component of the arrow vibration was investigated using the model of a flexible rod, which is loaded with the longitudinal force of inertia and the driving force of the bowstring, accelerating the rod. The inertial and elastic properties of the bow (closed kinematic chain) in this model are taken into account by an elementary mechanical oscillator attached to the bowstring socket [3].

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The corresponding mathematical model has been represented by a differential equation with variable coefficients and inhomogeneous boundary conditions. The algebration of the boundary value problem is carried out by constructing an iterative process by representing the Eigen forms of the rod by a polynomial with the subsequent implementation of the solution of the problem using a computer program in a high-level programming language.

The joint movement of an arrow with a bow in the projection on the main (vertical) plane was investigated using the model of a hinged kinematic chain with absolutely non-deformable rods-links . In this case, the mathematical model was represented by a system of differential equations with variable coefficients and initial conditions. The solution of the corresponding Cauchy problem was carried out by the Runge-Kutta method using the "Mathematica Wolfram" computer mathematics system (CMS) [4, 5].

The ***purpose*** of this work is to create a mechanical and mathematical model of a complex open kinematic chain for the analysis of vibration processes in the "bow-arrow" system, taking into account the bending deformations of the arrow.

General approach to modeling. On the basis of generalization of the described models, it becomes possible to achieve the outlined goal. The solution of these problems has been brought to the level of engineering methodology, although the use of SCM, other things being equal, looks more convenient and practical. Therefore, a mandatory requirement for the model to be created is its suitability for implementation in one of the SCMs generally available for engineering practice. Judging by the papers, a mathematical model for the analysis of dynamic processes in a complex open kinematic chain can be a system of partial differential equations with partial derivatives of time and longitudinal coordinate of the rod. Although there are ample opportunities for solving differential equations in modern commercial computer packages, we have not found standard functions for solving this initial-boundary value problem in the well-known SCMs.

On the other hand, the mechanical-mathematical model of dynamic longitudinal bending and rod vibration can be reduced to a system of simple differential equations by specifying a hypothetical form (or several forms) by functions that correspond to the boundary conditions (or some of them) and are qualitatively similar to the Eigen forms, which can be used to describe this

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dynamic process approximately and in general. A necessary condition for success with this approach is a successful choice of analytical functions from which hypothetical forms are "constructed", and the criterion for their similarity to real forms is the proximity of hypothetical values of natural frequencies to the corresponding real values. Thus, the problem of the dynamics of a complex open kinematic chain can be reduced to the Cauchy problem in the form of a system of ordinary differential equations with initial conditions [6,7].

Let us consider such a problem on the example of the dynamics of the "bow-arrow" system, which can be modeled by a connecting rod-rocker kinematic chain close to a symmetrical relatively open link-rod (boom) (Fig. 1). The rocker arm (bow handle – 1) is hinged to the rack (shooter's hands – 0). At the ends, connecting rods are attached to the rocker with elastic hinges (bow arms – 2, 3), to which, in turn, the branches of the bowstring are attached – an elastic stretched thread.

Since the branches of the bowstring are stretched throughout the operation of the mechanism (bow), they can be considered as connecting rods (4, 5). In the socket of the bowstring, an open link (arrow – 6) is hinged, which "complicates" and at the same time "opens" the chain. The asymmetry of the kinematic chain is due to the fact that the arrow and the arm holding the bow must be in different places – the hand holds the bow handle in the middle, and the arrow is slightly higher.

Symmetric design model. Since the asymmetry of the chain is small (3–4%), the corresponding symmetrical scheme of the kinematic chain can be adopted for the analysis of hypothetical shapes of bending oscillations of the boom with an accuracy acceptable for engineering calculations. Since the bending deformations of the boom do not exceed 5% of its length, we adopt a geometrically linear model of transverse displacements in the kinematic chain.

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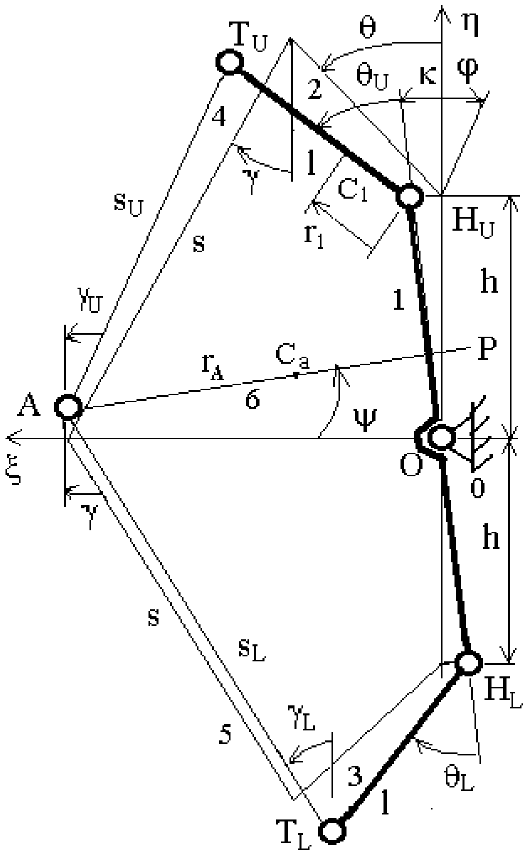

Fig.1. Symmetric design model of a complex kinematic chain.

The geometric and force parameters of the neutral position of the chain are as follows (see Fig. 1):

$$h + l\cos\theta = s\cos\gamma;$$

$$l\sin\theta + s\sin\gamma = \xi_A; \qquad (1)$$

$$c_s(s - s_0)l\sin(\theta + \gamma) = c_l(\theta + \varphi),$$

where c_l is virtual stiffness of the riser; s_0 is length of the string in a free situation; φ is angle of the riser in the free situation; $\xi O \eta$ is reference bogy system fixed to the pivot point of the handle (0). Other parameters are clear on the scheme model.

At a small deviation from the neutral position, the corresponding ratios for the upper (U) and lower (L) parts of the kinematic chain are as follows:

$$h + l\cos(\theta_{U/L} \pm \kappa) = s_{U/L}\cos\gamma_{U/L} \pm \eta_A;$$

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$$\pm h\kappa + l\sin(\theta_{U/L} \pm \kappa) + s_{U/L}\sin\gamma_{U/L} = \xi_A; \qquad (2)$$

$$c_s(s_{U/L} - s_0)l\sin(\theta_{U/L} + \gamma_{U/L}) = c_l(\theta_{U/L} + \varphi),$$

where in double arithmetic signs, the upper one refers to the ratios for the upper half of the chain, and the lower – refers to the lower one.

Let's connect the geometric parameters of the neutral and offset positions of the chain:

$$\theta_{U/L} = \theta + \Delta\theta_{U/L};$$

$$\gamma_{U/L} = \gamma + \Delta\gamma_{U/L}; \qquad (3)$$

$$s_{U/L} = s + \Delta s_{U/L},$$

where the letter Δ denotes small displacements and deformations. After substituting (3) into (2) and transforming (1), we get a system of three linear with respect to the sums $(\Delta\theta_U + \Delta\theta_L)$, $(\Delta\gamma_U + \Delta\gamma_L)$, $(\Delta s_U + \Delta s_L)$ of the homogeneous algebraic equations as follows:

$$l(\Delta\theta_U + \Delta\theta_L)\cos\theta + s(\Delta\gamma_U + \Delta\gamma_L)\cos\gamma + (\Delta s_U + \Delta s_L)\sin\gamma = 0;$$

$$l(\Delta\theta_U + \Delta\theta_L)\sin\theta - s(\Delta\gamma_U + \Delta\gamma_L)\sin\gamma + (\Delta s_U + \Delta s_L)\cos\gamma = 0; \qquad (4)$$

$$[c_s(s - s_0)l\cos(\theta + \gamma) - c_l](\Delta\theta_U + \Delta\theta_L) + c_s(s - s_0)l\cos(\theta + \gamma)(\Delta\gamma_U + \Delta\gamma_L) + \\ + c_s l\sin(\theta + \gamma)(\Delta s_U + \Delta s_L) = 0.$$

The determinant of this system in real correlations of characteristics of modern sports bows

$$\frac{s_0}{l} < 2; \ \frac{s - s_0}{s_0} < 0{,}02; \ 20^\circ < \theta + \gamma < 180^\circ,$$

$$c_s(s - s_0)l^2\cos^2(\theta + \gamma) + c_l s + c_s l^2 s\sin^2(\theta + \gamma) \times \left[1 - \frac{s_0}{l}\frac{s - s_0}{s_0}\frac{\cos(\theta + \gamma)}{\sin^2(\theta + \gamma)}\right] > 0.$$

As a result, the system (4) has only zero solutions:

$$\Delta\theta_U + \Delta\theta_L = 0; \ \Delta\gamma_U + \Delta\gamma_L = 0; \ \Delta s_U + \Delta s_L = 0.$$

So, in the future, when studying the small transverse oscillations of a symmetrical kinematic chain, we can assume:

$$\Delta\theta_U \equiv -\Delta\theta_L = \tau; \quad \Delta\gamma_U \equiv -\Delta\gamma_L; \quad \Delta s_U \equiv -\Delta s_L; \quad \Delta\theta'_U \equiv -\Delta\theta'_L = \tau'. \quad (5)$$

As a mathematical model of these oscillations, it is expedient to take the Lagrange equation of the second kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} = 0, \quad (6)$$

where T and P are the kinetic and potential energy of the system, respectively, q_i – generalized coordinates; t – time; $(') \equiv \partial/\partial t$ dash indicates a partial derivative of time. Taking into account equations (5), the expressions of energy have been derived [1]:

$$T = \frac{1}{2}\begin{Bmatrix} m_A\eta_A'^2 + \left(I_H + 2m_l h^2\right)\kappa'^2 + 2I_l\left(\tau' + \kappa'\right)^2 + \\ 4m_l r_l h\kappa'\left(\tau' + \kappa'\right)\left[\cos\theta - \left(\tau + \kappa\right)\sin\theta\right] \end{Bmatrix};$$

$$P = P_l + P_s, \quad (7)$$

where m_A is virtual mass of a string and an arrow; I_H is moment of inertia of the handle, stabilizers and a sign relatively a pivot point; I_l – moment of inertia of the riser; r_l – a distance of a center of mass; $P_l = \frac{1}{2}c_l\left[\left(\theta_U + \varphi\right)^2 + \left(\theta_L + \varphi\right)^2 - 2\left(\theta + \varphi\right)^2\right] = c_l\tau^2$ – change in the potential energy of the bowstring and is: $\Delta s = -\left[l\tau\sin\left(\theta + \gamma\right) + \left(\xi_A\kappa + \eta_A\right)\cos\gamma\right]$.

Assigning generalized coordinates $\left(q_i \equiv \kappa, \tau, \eta_A\right)$, let us write down the differential equations of transverse oscillations of the kinematic chain:

$$I_\kappa\kappa'' + I_\tau\tau'' + c_s\Delta s\xi_A\cos\gamma = 0; \quad \tfrac{1}{2}m_A\eta''_A + c_s\Delta s\cos\gamma = 0, \quad (8)$$

Where $I_\kappa = \frac{1}{2}I_H + m_l h_l^2 + I_l + 2m_l h_l r_l\cos\theta$;

$I_l\tau'' + c_l\tau + c_s l\Delta s\sin\left(\theta + \gamma\right) + I_\tau\kappa'' = 0$, $I_\tau = I_l + mh_l r_l\cos\theta$.

Geometric parameters of the neutral position of the chain $\left(\xi_A, \theta, \gamma\right)$, we have determined from the system of equations (1).

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Natural Frequencies ω and forms of oscillations of the system have been determined by substituting solutions of equation (8) in the determinant:

$$\begin{vmatrix} b_{\kappa\kappa} & b_{\kappa\tau} & b_{\kappa\eta} \\ * & b_{\tau\tau} & b_{\tau\eta} \\ * & * & b_{\eta\eta} \end{vmatrix} = 0, \qquad (9)$$

where $b_{\kappa\kappa} = c_s(\xi_A \cos\gamma)^2 - \omega^2 I_\kappa$; $b_{\kappa\eta} = c_s \xi_A \cos^2\gamma$; $b_{\kappa\tau} = c_s l \sin(\theta+\gamma)\xi_A \cos\gamma - \omega^2 I_\tau$; $b_{\tau\tau} = c_s[l\sin(\theta+\gamma)]^2 + c_l - \omega^2 I_l$; $b_{\eta\eta} = c_s \cos^2\gamma - \frac{1}{2} m_A \omega^2$.

A preliminary analysis of the vibrations in the circuit under consideration can be carried out assuming that the outer link is a non-deformable rod, which is, neglecting its compliance, which is significantly less than the compliance of the elastic joints H_U and H_L reduced to the axis of the hinge A (see Fig. 1). We take into account the inertial properties of the outer link by bringing them to the axis of the hinge. The mass of the arrow reduced to the bowstring socket (the hinge connecting the outer link to the chain) is determined from the system of equations of the amount of motion and the moment of the amount of movement:

$$\int_0^{l_a} \mu(z)\eta'_a(z)dz = m_A \eta'_A; \int_0^{l_a} \mu(z)\eta'_a(z) z dz = 0, \qquad (10)$$

where $\mu(z)$ – distributed mass of a string; m_a – its whole mass; l_a – length of a string; $\eta_a(z) = \eta_A + \psi z$ – lateral displacements of a free chain as a solid shift. By substituting this form into equation (9), we obtain the expression for the reduced mass: $m_A = \dfrac{m_a r_a^2}{r_A^2 + r_a^2}$, where $r_A = \dfrac{\int_0^{l_a} \mu(z) z dz}{m_a}$ is a distance to the center of mass of a string; $r_a = \sqrt{\dfrac{\int_0^{l_a} \mu(z)(z - r_A)^2 dz}{m_a}}$ – radius of inertia of a string.

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For example, a modern sport bow of common size could be modeled by the kinematic chain with mechanical parameters as follows: $I_K = 0.671$ kgm^2; $m_l = 0.0953$ kg; $h_l = 0.4343$ m; $I_l = 0.006344$ kgm^2; $r_l = 0.2165$ m; $l = 0.48$ m; $c_l = 120.78$ m; $c_s = 14000$ m; $s_0 = 0.85$ m; $\xi_A = l_a = 0.7$ m; $\theta = 0.7608$ rad; $\gamma = 0.4410$ rad; $m_a = 0.0091$ kg. By substituting these parameters into equation (9), we obtain the values of the Eigen circular frequencies $\omega_1 = 132.293$ s^{-1} i $\omega_2 = 1730$ s^{-1}. If you neglect the malleability of the bowstring $(c_s = \infty)$, the circuit will have only one natural frequency smaller than 0.05%. The solution was obtained using standard subroutines from the SCM MathCAD package.

Model of a chain. Dynamic processes in the complex open kinematic chain are accompanied by intense vibrations, in particular, bending oscillations of the outer link, associated with the loss of dynamic stability in the transverse direction [3]. The engineering technique for analyzing these processes can be based on a model of dynamic longitudinal bending of a rod describing an outer link, with a simple mechanical oscillator attached to it, describing the closed part of the kinematic chain.

The stiffness of connecting rods in kinematic chains of this type under consideration is generally significantly higher than the stiffness of elastic hinges, therefore, in the future, for the analysis of vibration processes, we can adopt a model of a simple linear oscillator with a flexible rod hinged attached to it (Fig. 2). The vibrations of the free rod may be considered as decomposed according to their own forms, but the predominant contribution usually belongs to the form of the fundamental tone or the first two forms. These proper shapes have one, two, or three nodes.

Eigen forms are written using Krylov functions, but with accuracy acceptable for engineering calculations, instead of exact expressions of Eigen forms, hypothetical forms can be adopted, which describe these forms quite accurately and allow calculating the corresponding Eigen frequencies with an acceptable error [3].

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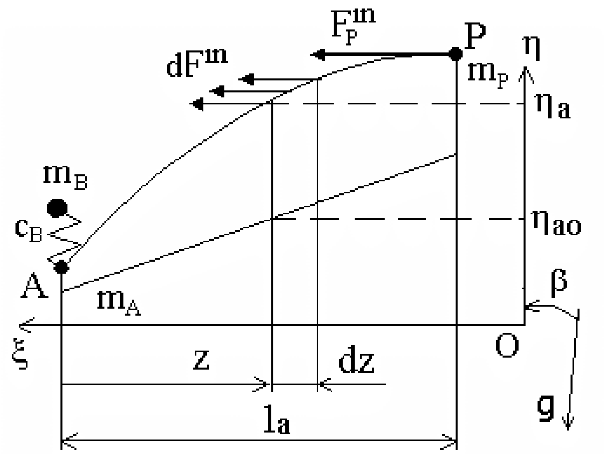

Fig.2. A virtual scheme model of vibration in the open kinematic chain: g is gravity acceleration.

To "construct" hypothetical shapes, we use a linear function and a sine wave. As a model of a hypothetical form of vibration of the outer link, it is advisable to use a sinusoid half-wave shifted in the plane of the kinematic chain, which makes it possible to obtain the values of natural frequencies and amplitudes of vibrations with accuracy acceptable in engineering calculations:

$$\eta_{a3} = \eta_A + \psi z + f_2 \sin\frac{\pi z}{l_a} + f_3 \sin\frac{2\pi z}{l_a}, \qquad (11)$$

where f_2, f_3 - amplitudes of components of bend deformations.

Potential and kinetic energy in the system have been derived as follows:

$$P = \frac{1}{2} c_B (\eta_B - \eta_A)^2 \frac{1}{2} \int_0^{l_2} \varepsilon(z_a) \left(\frac{\partial^2 \eta_a}{\partial z_a^2} \right)^2 dz_a +$$

$$+ \frac{1}{2} \xi''_A \int_0^{l_a} \mu(z_a) \left[\int_0^{z_a} \left(\frac{\partial \eta_a}{\partial z_a} \right)^2 d\chi + \left(\frac{\partial \eta_a}{\partial z_a} \right)^2_{z_a = l_a} \right] dz_a;$$

$$T = \frac{1}{2} m_B \eta_B'^2 + \frac{1}{2} \int_0^{l_a} \mu_a(z_a) \eta_a'^2 dz_a,$$

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where c_B – virtual stiffness of risers (internal chain); m_B – virtual mass of a bow; $\varepsilon(z_a)$ – distributed stiffness of an arrow (external chain). By substituting the expression for the form (11) in the last two expressions, we obtain the expressions of the energy of the system under transverse oscillations:

$$P=\frac{1}{2}c_B(\eta_B-\eta_A)^2+\frac{\pi^4\varepsilon}{4l_a^3}f_2^2+\frac{4\pi^4\varepsilon}{l_a^3}f_3^2+$$

$$+\frac{1}{2}\xi_A''\left[\begin{array}{l} m_{sh}\left(\frac{l_a}{2}\psi^2+\frac{4}{\pi}\psi f_2+\frac{\pi^2}{4l_a}f_2^2+\frac{\pi^2}{l_a}f_3^2+\frac{16}{3l_a}f_2 f_3\right) \\ +m_P\left(l_a\psi^2+\frac{\pi^2}{2l_a}f_2^2+\frac{2\pi^2}{l_a}f_3^2\right)\end{array}\right];$$

$$T=\frac{1}{2}\left[\begin{array}{l} m_{sh}\left(\begin{array}{l}\eta_A'^2+\frac{l_a^2}{3}\psi'^2+\frac{f_2'^2}{2}+\frac{f_3'^2}{2}- \\ \frac{l_a}{\pi}\psi' f_3'+l_a\eta_A'\psi'+\frac{4}{\pi}\eta_A' f_2'+\frac{2}{\pi}l_a\psi' f_2'\end{array}\right) \\ +m_A\eta_A'^2+m_P(\eta_A'+l_a\psi')^2+m_B\eta_B'^2\end{array}\right], \qquad (12)$$

where m_{sh} – the mass of the boom shaft (i.e. without the mass of the tip and shank).

Assigning generalized coordinates $q_i \equiv \eta_A, \eta_B, \psi, f_2, f_3$ and substituting the energy expressions into the Lagrange equation (6), we obtain a system of equations of transverse oscillations of the chain:

$$(m_{sh}+m_A+m_P)\eta_A''+\left(\frac{m_{sh}}{2}+m_P\right)l_a\psi''+\frac{2}{\pi}m_{sh}f_2''-c_B(\eta_B-\eta_A)=0;$$

$$m_B\eta_B''+c_B(\eta_B-\eta_A)=0;$$

$$\left(\frac{m_{sh}}{3}+m_P\right)l_a\psi''+\left(\frac{m_{sh}}{2}+m_P\right)\eta_A''+\frac{m_{sh}}{\pi}f_2''-\frac{m_{sh}}{2\pi}f_3''+$$

$$+\left[\left(\frac{m_{sh}}{2}+m_P\right)l_a\psi+\frac{2}{\pi}m_{sh}f_2\right]\frac{\xi_A''}{l_a}=0;$$

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$$\frac{m_{sh}}{2} f_2'' + \frac{2}{\pi} m_{sh} \eta_A'' + \frac{m_{sh}}{\pi} l_a \psi'' + \frac{\pi^4 \varepsilon}{2 l_a^3} f_2 +$$
$$+\left[\frac{\pi^2}{2}\left(\frac{m_{sh}}{2} + m_P\right) f_2 + \frac{2}{\pi} m_{sh} l_a \psi + \frac{8}{3} m_{sh} f_3\right] \frac{\xi_A''}{l_a} = 0; \quad (13)$$

$$\frac{m_{sh}}{2} f_3'' - \frac{m_{sh}}{2\pi} l_a \psi'' + \frac{8\pi^4 \varepsilon}{l_a^3} f_3 +$$
$$+\left[\pi^2 m_{sh} f_3 + \frac{8}{3} m_{sh} f_2 + 2\pi^2 m_P f_3\right] \frac{\xi_A''}{l_a} = 0.$$

like (9), the main determinant of the system of equations (13) linear with respect to amplitudes is equated to zero:

$$\begin{vmatrix} b_{00} & b_{01} & b_{02} & b_{03} & b_{04} \\ * & b_{11} & b_{12} & b_{13} & b_{14} \\ * & * & b_{22} & b_{23} & b_{24} \\ * & * & * & b_{33} & b_{34} \\ * & * & * & * & b_{44} \end{vmatrix} = 0, \quad (14)$$

where $b_{00} = \nu - (1 + \varsigma_A + \varsigma_B)\Omega^2$; $b_{01} = -\nu$; $b_{02} = -\left(\frac{1}{2} + \varsigma_P\right)\Omega^2$;

$b_{03} = -\frac{2}{\pi}\Omega^2$; $b_{04} = b_{12} = b_{13} = b_{14} = 0$; $b_{11} = \nu - \varsigma_B \Omega^2$;

$b_{22} = -\left(\frac{1}{3} + \varsigma_P\right)\Omega^2 - \left(\frac{1}{2} + \varsigma_P\right)\Phi$; $b_{23} = -\frac{\Omega^2 + 2\Phi}{\pi}$; $b_{24} = \frac{\Omega^2}{2\pi}$;

$b_{33} = \frac{1}{2}\left\{\pi^2\left[4\pi^2 - \left(\frac{1}{2} + \varsigma_P\right)\Phi\right] - \Omega^2\right\}$; $b_{34} = -\frac{8}{3}\Phi$;

$b_{44} = 2\pi^2\left[4\pi^2 - \left(\frac{1}{2} + \varsigma_P\right)\Phi\right] - \frac{\Omega^2}{2}$; $\varsigma_A = \frac{m_A}{m_{sh}}$; $\varsigma_P = \frac{m_P}{m_{sh}}$; $\varsigma_B = \frac{m_B}{m_{sh}}$;

$\Omega^2 = \omega^2 \frac{m_{sh} l_a^3}{\varepsilon}$; $\Phi = \frac{-\xi_A'' m_{sh} l_a^2}{\varepsilon}$; $\upsilon = \frac{c_B l_a^3}{\varepsilon}$.

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To identify the optimal (from the point of view of simplicity and sufficient accuracy in engineering calculations) model of the vibration form of the external link, let us consider simplified versions of the hypothetical shape model (11). One of them describes the movements of the free link as a non-deformable rod:

$$\eta_{a1} = \eta_A + \psi z, \tag{15}$$

and the second – as a flexible rod with two knots of bending deformation:

$$\eta_{a2} = \eta_A + \psi z + f_2 \sin\frac{\pi z}{l_a}. \tag{16}$$

Natural Frequencies An arrow loses stability at critical values of longitudinal acceleration (the "exact" solution [3]). The approximate solution obtained using the hypothetical form (16) gives Φ_1 = 15.42 (error 4.3%), and using the hypothetical form (11) – Φ_1 = 14.56 (1.6%) and Φ_2 = 65.68 (16.3%). The hypothetical form (15) does not describe the loss of stability at accelerations Φ_1 and Φ_2, and the form (16) describes the loss of stability at only one value of acceleration (Φ_1). As can be seen from the data (Table 1) and graphs (Fig. 3), all three hypothetical forms give an exact ("degenerate") solution for $\Phi_0 = 0$.

Similar to the previous problem (symmetric circuit diagram), in this case, the solution was obtained using standard subroutines from the SCM MathCAD package. Thus, the proposed approach to modeling vibration processes in a complex open kinematic chain allows to implement the solution by numerical methods from a set of standard subroutines of widely available SCM packages. Thus, the mechanical-mathematical model of a complex open kinematic chain allows us to study the complex dynamic processes of an impulse nature that characterize the behavior of the "arrow-bow" system, in particular, vibrations and loss of stability of the arrow during its joint movement with the bow.

A graphical representation of the dependencies of the system’s natural frequencies on the magnitude of the acceleration of the longitudinal motion of the outer link (Fig. 3) allows us to establish that with an increase in acceleration, the natural frequencies decrease.

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Table 1. Values of natural frequencies of the kinematic chain

Model	Φ – longitudinal acceleration		
	0	10	20
[3]	0	5.29j	12.41j
(15)	0	4.90j (-7.4%)*	7.55j (39.0%)
(16)	0	5.24j (-0.5%)	11.46 (-7.5%)
(11)	0	5.30j (0.2%)	12.65 (2.0%)
[3]	6.63	5.43	5.83j
(15)	7.01 (5.7%)	5.90 (9.8%)	–
(16)	6.65 (0.3%)	5.52 (1.76%)	5.30j (-9.1%)
(11)	6.64 (0.2%)	5.44 (0.2%)	5.69j (-2.4%)
[3]	15.66	8.63	5.46
(15)	–	–	5.42 (-0.7%)
(16)	15.70 (0.3%)	8.81 (2.1%)	5.47 (0.2%)
(11)	15.69 (0.2%)	8.55 (0.9%)	5.46 (0.0%)
[3]	45.96	41.16	35.80
(11)	46.01 (0.1%)	41.25 (0.2%)	36.17 (1.0%)

*Note: Errors relative to the exact solutions [3].

One of the eigen forms of vibrations in the presence of acceleration reflects the loss of stability of the free link, which is manifested in a monotonous increase in the angle of rotation of its axis relative to the direction of longitudinal movement. At the value of acceleration corresponding to the value of the dimensionless parameter Φ = 15, there is a loss of stability of the free link along the first bending shape, which has one node, and at Φ = 65 – along the second bending shape, which has two nodes.

In the range of small and medium accelerations (Φ < 10) the accuracy of solutions for natural frequencies when using models of hypothetical shapes with two (16) or three (11) nodes, taking into account the requirements of engineering calculations, is almost the same (error within 2%). In the range of big and medium accelerations (Φ > 20), achieving of such accuracy is possible only when using a hypothetical shape model with three nodes (Table 1).

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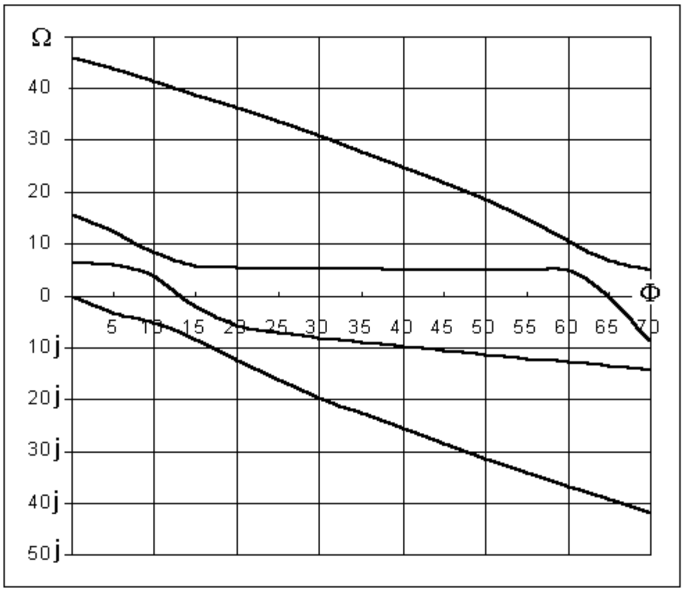

Fig.3. Dependence of the natural frequencies of the circuit on the acceleration of the longitudinal motion of the free link: j = $(-1)^{1/2}$ – imaginary unit.

The use of a hypothetical shape model with one node (15) leads to errors higher than 5% and can be recommended for a preliminary analysis of vibration processes in a circuit when the bending component of vibration can be neglected.

Conclusion. Dynamic processes in a complex open kinematic chain are accompanied by intense vibrations, in particular, bending oscillations of the outer link, associated with the loss of dynamic stability in the transverse direction. The engineering technique for analyzing these processes can be based on a model of dynamic longitudinal bending of a rod describing an outer link, with a simple mechanical oscillator attached to it, describing the closed part of the kinematic chain.

As a model of a hypothetical form of vibration of the external link, it is advisable to use a sinusoid half-wave shifted in the plane of the kinematic chain, which allows obtaining the values of natural frequencies and amplitudes of vibrations with accuracy acceptable in engineering calculations. The mechanical-mathematical model of a complex open kinematic chain allows us to study complex dynamic processes of an impulse nature that characterize the behavior of the "arrow-bow" system, in particular, vibrations and loss of stability of the arrow during its joint movement with the bow. The proposed approach to simulation of vibration processes in a complex open kinematic chain allows carrying out the implementation by numerical methods from a set of standard subroutines of widely available SCM packages.

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Статтю представив д-р техн. наук, проф. Національного технічного університету "Харківський політехнічний інститут" С.Ю. Леонов.

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Вібрації у відкритому складеному кінематичному ланцюзі // І.П. Заневський // Вісник НТУ "ХПІ". Серія: Інформатика та моделювання. – Харків: НТУ "ХПІ". – 2025. – № 2 (14). – С. 114 – 129.

Метою дослідження було створення моделі вібраційних процесів у складному розімкнутому кінематичному ланцюзі із зовнішньою ланкою. Запропоновано механічну та математичну модель поперечних коливань системи під час прискореного руху зовнішньої ланки. Отримано кореляцію між поздовжнім прискоренням та власними частотами. Надано рекомендації щодо визначення віртуальних форм вібрації зовнішньої ланки. Моделі та методи адаптовані для реалізації інженерним методом з використанням відомих математичних САПР. Приклад механіко-математичної моделі складного розімкнутого кінематичного ланцюга дозволяє досліджувати складні динамічні процеси імпульсного характеру, що характеризують поведінку системи "стріла-лук", зокрема, вібрації та втрату стійкості стріли під час її спільного руху з луком. Запропонований підхід до моделювання вібраційних процесів у складному розімкнутому кінематичному ланцюзі дозволяє здійснювати реалізацію числовими методами з набору стандартних підпрограм широкодоступних пакетів SCM. Іл.: 3. Бібліогр.: 7 назв.

Ключові слова: модель, вібрація, складна, кінематичний ланцюг, власна форма

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Vibrations in the open compound kinematical chain // I.P. Zanevskyy // Herald of the National Technical University "KhPI". Series of "Informatics and Modeling". – Kharkov: NTU "KhPI". – 2025. – № 2 (14). – P. 114 – 129.

The aim of the research was to create a model of vibration processes in the compound open kinematical chain with an external link. A mechanical and mathematical model of the lateral oscillations of the system during the external link accelerated motion is proposed. Correlation between longitudinal acceleration and natural frequencies are obtained. There are recommendations regarding determinations of the virtual forms of external link's vibration. The models and methods have been adapted for realization in the engineering method using well-known mathematical CAD systems. An example of the mechanical-mathematical model of a complex open kinematic chain allows us to study complex dynamic processes of the impulse nature that characterize the behavior of the "arrow-bow" system, in particular, vibrations and loss of stability of the arrow during its joint movement with the bow. The proposed approach to simulation of vibration processes in a complex open kinematic chain allows carrying out the implementation by numerical methods from a set of standard subroutines of widely available SCM packages. Il.: 3. Ref.: 7 Items

Keywords: model, vibration, compound, kinematic chain, Eigenform