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FUZZY MULTI-INDEX NONLINEAR TRANSPORTATION PROBLEMS

Object of research: a nonlinear separable transportation problem with fuzzily specified input data is considered. The conditional optimization problem is transformed into an unconstrained one. The solution process employs a composite criterion with its value dependent on the membership functions of the fuzzy parameters. The computational optimization procedure is organized in accordance with the Nelder-Mead method.
Keywords: nonlinear separable optimization; fuzzy input data.

Introduction. Numerous problems in engineering, economics, military science, sociology, and other fields lead to a uniform model consisting in finding a set of two-index decision variables that minimize an additive nonlinear separable objective function subject to a system of linear constraints on the variables. For example, consider the problem of minimizing the total cost of transporting a homogeneous product from a given set of m suppliers who have the product available in amounts a_1, a_2, …, a_m, to a system of n consumers requesting this product in quantities b_1, b_2,…, b_n. The cost of transporting x_{ij} units of the product from the i - th supplier to the j -th customer is determined by the function $\phi_{ij}(x_{ij})$. In this case, the formal model of the problem can be written as follows: find the set $X = \{x_{ij}\}$, that minimizes the objective function

$$F(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \phi_{ij}(x_{ij}) \quad (1)$$

And satisfying the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\ldots,m, \quad (2)$$

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$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,...,n, \tag{3}$$

$$x_{ij} \geq 0, i = 1,2,...,m, j = 1,2,...,n. \tag{4}$$

The resulting problems are successfully solved using a methodology based on decomposing the original two-index problem into a set of single-index problems [1]. At the same time, significant difficulties arise when the parameters of the objective function (1) cannot, for objective reasons, be precisely determined and, accordingly, must be described, for example, in terms of fuzzy mathematics [2 – 7]. Let us consider a methodology for solving such problems in the special case where the parameters of the objective function in a two-index problem are fuzzily specified [8].

Data and methods. We formulate a mathematical programming problem with an additive separable nonlinear two-index objective function with fuzzy parameters and linear transportation-type constraints (2) – (4) [9].

Let, for definiteness, the components of the objective function (1) be of the following form

$$\phi_{ij}(x_{ij}) = c_{ij}x_{ij}^{p}, p \in (0,1),\ i = 1,2,...,m, j = 1,2,...,n,$$

where c_{ij} parameters are fuzzy numbers with the membership function

$$\mu(c_{ij}) = exp\left\{-\frac{\left(c_{ij}-c_{ij}^{(0)}\right)^2}{2\sigma_{ij}^2}\right\}. \tag{5}$$

In this connection the objective function takes the following form

$$F(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}^{p}. \tag{6}$$

The problem is formulated as follows: find the set$X = \{x_{ij}\}$
that minimizes (6), taking into account (5), and satisfies the constraints (2) – (4). The traditional methodology for solving such problems consists of transforming the original fuzzy mathematical programming problem into a crisp one as follows [4–6]. Let us choose a number $\alpha \in (0;1)$such that the inequality

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$\sup_{c_{ij}} \mu(c_{ij}) \geq \alpha$holds for all membership functions of the fuzzy parameters of the objective function.

Then, in order to obtain a solution X whose degree of non-dominance is at least α, it is sufficient to solve the following mathematical programming problem: find the sets $X = \{x_{ij}\}$ and $C = \{c_{ij}\}$ that minimize (6), satisfy the constraints (2) – (4) and, in addition, the constraints

$$\mu(c_{ij}) = exp\left\{-\frac{\left(c_{ij}-c_{ij}^{(0)}\right)^2}{2\sigma_{ij}^2}\right\} \geq \alpha, i = 1,2,\dots,m, j = 1,2,\dots,n. \quad (7)$$

The drawbacks of this approach are fairly obvious: the increasing dimensionality of the problem, possible distortion of the original problem, and the lack of definiteness in choosing the parameter α. In this regard, an alternative procedure for solving the problem (2) – (6) is considered.

We formulate the following composite objective function. First, we introduce

$$G_i(x) = a_i - \sum_{j=1}^{n} x_{ij}, i = 1,2,\dots,m,$$
$$H_j(x) = b_j - \sum_{i=1}^{m} x_{ij}, j = 1,2,\dots,n,$$
$$M(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \mu(x_{ij})$$

and then we define

$$W(x) = \frac{F(x)+\sum_{i=1}^{m} G_i^2(x)+\sum_{j=1}^{n} H_j^2(x)}{M(x)}. \quad (8)$$

It is clear that reducing the values of the terms in the numerator of W(x) and increasing the values in the denominator will lead to a decrease in (8), as required.

To optimize the proposed objective function, it is suggested to use an efficient optimization procedure based on the Nelder-Mead method, which is described in detail in the literature source [10].

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Conclusions. A method for solving nonlinear separable transportation problems has been developed and implemented. The computational procedure has been extended to the case where the input data are fuzzily specified.

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Об'єкт дослідження: розглянуто нелінійну сепарабельне транспортну задачу з нечітко заданими вихідними даними. Задачу умовної оптимізації перетворено на безумовну. При вирішенні використано комплексний критерій, значення якого залежить від функцій належності нечітких параметрів задачі. Обчислювальну процедуру оптимізації організовано відповідно до методу Нелдера-Міда.
Бібліогр.: 10 назв.

Ключові слова: нелінійна сепарабельна оптимізація, нечіткі вихідні дані.

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