Вісник Національного технічного університету "ХПІ", 2024, № 1–2 (11–12)
ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

UDC 004.94 DOI: 10.20998/2411-0558.2024.01.01

L. RASKIN, Dr.Sci.Tech., Prof., NTU "KhPI", Kharkiv,
L. SUKHOMLYN, CSc.Tech., Assoc. Prof., KrNU, Kremenchuk,
A. HATUNOV, master, PhD, NTU "KhPI", Kharkiv,
S. ANDRIIENKO, master, PhD, NTU "KhPI", Kharkiv

OPTIMAL DISTRIBUTION OF LIMITED RESOURCE IN A CONTEXT OF UNCERTAINTY

Object of research: methods of rational distribution of limited resource in a context of uncertainty. It is assumed that the system consists of a set of independently functioning elements, and effectiveness of these elements depends on the level of investment. The complexity of solving the problem using traditional methods depends on and is determined by the type of analytical description of production functions of the system elements. Significant difficulties arise if parameters of production functions are uncertain quantities, specified, for example, in terms of fuzzy mathematics. This circumstance emphasizes the relevance of studying approaches to solving the set problem of resource distribution for the case when its parameters are not clearly defined, and this circumstance also determines the goal of the work. Problems arising from the goal consist in developing mathematical models and methods for rational resource distribution for the main types of production functions with their parameters being fuzzy numbers of (L-R) type. To solve these problems, a method is proposed that transforms the original fuzzy problems into clear ones that can be solved using standard constrained optimization technologies. An important result of the research consists in the fact that the proposed method is universal, that is, it is applied in the same way to solve problems of resource distribution in systems wherein production functions of the elements can be of arbitrary nature.

Keywords: rational resource distribution; fuzzy parameters; fuzzy optimization; fuzzy mathematics.

Introduction. The problem of rational distribution of a limited resource belongs to the class of nonlinear mathematical programming problems [1]. Constructional features of such problems determine their belonging to constrained optimization problems [2, 3]. Let us consider the specific problem of distributing a one-dimensional resource in a multi-element production system [4].

Let us introduce a vector $x = (x_1, x_2, \dots, x_n)$, that determines the distribution of the resource among the elements of the system, and a set of

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$f_j(x_j) = a_j x_j^{\alpha}, j = 1,2,\dots,n,$ one-parameter production functions of the system elements.

Let us set the multiplicative production function of the system, which determines the criterion for the efficiency of resource distribution

$$. \; F(x) = \prod_{j=1}^{n} a_j x_j^{\alpha} = \prod_{j=1}^{n} \left((a_j) \prod_{j=1}^{n} x_j^{\alpha} \right) = a_0 \prod_{j=1}^{n} x_j^{\alpha}. \qquad (1)$$

Let us introduce a limitation on the amount of total resource consumed

$$\sum_{j=1}^{n} x_j = C. \qquad (2)$$

We are going to find the unknown vector $X = (x_1, x_2, \dots, x_n)$ by means of using the method of Lagrange undetermined multipliers. Let us introduce the Lagrange function

$$\phi(x, \lambda) = a_0 \prod_{j=1}^{n} x_j^{\alpha} - \lambda \left(\sum_{j=1}^{n} x_j - C \right). \qquad (3)$$

Next

$$\frac{\partial \phi(x,\lambda)}{\partial x_{j_0}} = a_0 \alpha \left(\prod_{j \neq j_0} x_j^{\alpha} \right) x_{j_0}^{\alpha-1} - \lambda = 0 \qquad (4)$$

or

$$\frac{a_0 \alpha}{x_{j_0}} \prod_{j=1}^{n} x_j^{\alpha} - \lambda = 0.$$

Hence

$$x_{j_0} = \frac{a_0 \alpha}{\lambda} \prod_{j=1}^{n} x_j^{\alpha}, \quad j_0 = 1,2,\dots,n. \qquad (5)$$

We are going to find the unknown value λ from the equation (2). Here we have the following

$$\sum_{j=1}^{n} x_j = \frac{1}{\lambda} n a_0 \alpha \prod_{j=1}^{n} x_j^{\alpha} = C,$$

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$$\frac{1}{\lambda} = \frac{C}{na_0\alpha\prod_{j=1}^{n} x_j^{\alpha}}. \qquad (6)$$

By means of substituting (6) into (5), we receive the following

$$x_j = \frac{C}{na_0\alpha\prod_{j=1}^{n} x_j^{\alpha}} \cdot \alpha_0\alpha\prod_{j=1}^{n} x_j^{\alpha} = \frac{C}{n}.$$

The triviality of the resulting solution can be explained by the multiplicativity of criterion (1) and the extreme simplicity of the production function. Let us now obtain a solution to this problem by introducing another, alternative option for constructing a criterion. Let it be as follows

$$F(x) = \sum_{j=1}^{n} f_j(x_j) = \sum_{j=1}^{n} a_j x_j^{\alpha}. \qquad (7)$$

Again, by means of using the method of Lagrange undetermined multipliers, we obtain the following

$$\phi(x,\lambda) = \sum_{j=1}^{n} a_j x_j^{\alpha} - \lambda(\sum_{j=1}^{n} x_j - C).$$

Next

$$\frac{\partial\phi(x,\lambda)}{\partial x_j} = a_j\alpha x_j^{\alpha-1} - \lambda = 0,$$

$$x_j^{\alpha-1} = \frac{\lambda}{a_j\alpha}, \quad j = 1,2,\ldots,n,$$

whence

$$x_j = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \cdot \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}.$$

We are going to find the unknown factor $\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}}$ by means of using restrictions (2):

$$\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} =$$

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$$= \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \cdot \sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} = C.$$

Hence

$$\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{C}{\sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}.$$

Then

$$x_j = \frac{C \cdot \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}} - \frac{C a_j^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n} a_j^{\frac{1}{1-\alpha}}}. \qquad (8)$$

Let it be, for example, as follows $\alpha = \frac{1}{2}$. Herewith we receive the following

$$x_j = C \frac{a_j^{\frac{1}{1-0,5}}}{\sum_{j=1}^{n} (a_j)^{\frac{1}{1-0,5}}} = C \cdot \frac{a_j^2}{\sum_{j=1}^{n} a_j^2}, j = 1,2,\dots,n.$$

Let us now solve this problem using a more informative analytical representation of the production function of the system elements and an additive criterion for its efficiency.

Let us introduce the following

$$f_j(x) = a_j x_j^{\alpha_j}.$$

Next

$$F(x) = \sum_{j=1}^{n} f_j(x_j) = \sum_{j=1}^{n} a_j x_j^{\alpha_j}.$$

In this case the Lagrange function has the following form:

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$$\phi(x,\lambda) = \sum_{j=1}^{n} a_j x_j^{\alpha_j} - \lambda\left(\sum_{j=1}^{n} x_j - C\right).$$

Next

$$\frac{\partial\phi(x,\lambda)}{\partial x_j} = a_j\alpha_j x_j^{\alpha_j-1} - \lambda = 0, \quad j = 1,2,\dots,n. \tag{9}$$

Hence

$$x_j^{\alpha_j-1} = \frac{\lambda}{a_j\alpha_j}, \quad x_j = \left(\frac{\lambda}{a_j\alpha_j}\right)^{\frac{1}{\alpha_j-1}} =$$

$$= (\lambda)^{\frac{1}{\alpha_j-1}} \cdot \left(\frac{1}{a_j\alpha_j}\right)^{\frac{1}{\alpha_j-1}}.$$

Let us find the factor $(\lambda)^{\frac{1}{\alpha_j-1}}$, by means of using (2) again.

$$\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} \left(\frac{\lambda}{a_j\alpha_j}\right)^{\frac{1}{\alpha_j-1}} = C.$$

The resulting equation (being non-linear relative to λ) can be solved numerically. Its simple analytical solution can be found in the special case when $\alpha_j = \alpha, j = 1,2,\dots,n$. Then relation (9) takes the following form:

$$x_j = \lambda^{\frac{1}{\alpha-1}} \left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}.$$

At the same time

$$\sum_{j=1}^{n} x_j = (\lambda)^{\frac{1}{\alpha-1}} \sum_{j=1}^{n} \left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}} = C,$$

whence

$$(\lambda)^{\frac{1}{\alpha-1}} = \frac{\mathrm{C}}{\sum_{j=1}^{n} \left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}}$$

and

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$$x_j = \frac{C\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n}\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}}, \quad j = 1,2,\dots,n. \tag{10}$$

Let us reduce (10) to a form more convenient for calculation:

$$x_j = \mathrm{C}\frac{(a_j\alpha)^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n}(a_j\alpha)^{\frac{1}{1-\alpha}}} = C\frac{a_j^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n}a_j^{\frac{1}{1-\alpha}}},$$

which (in view $\alpha_j = \alpha$,) coincides with (8).

Let us now consider the problem of two-dimensional resource distribution with one-parameter production functions of system elements. Let us introduce the production function of the system elements

$$f_j(x_1, x_2) = a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha};\ j = 1,2,\dots,n; \tag{11}$$

$$F(x_1, x_2) = \sum_{j=1}^{n} f_j(x_1, x_2) =$$

$$= \sum_{j=1}^{n}\left(a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha}\right). \tag{12}$$

The restrictions on a two-dimensional resource have the following form:

$$\sum_{j=1}^{n} x_{1j} = C_1, \tag{13}$$

$$\sum_{j=1}^{n} x_{2j} = C_2. \tag{14}$$

Next

$$\phi(x_1, x_2) =$$

$$\sum_{j=1}^{n}\left(a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha}\right) - \lambda_1\left(\sum_{j=1}^{n} x_{1j} - C_1\right) -$$

$$-\lambda_2\left(\sum_{j=1}^{n} x_{2j} - C_2\right),$$

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

$$\frac{\partial\phi(x_1,x_2)}{\partial x_{1j}} = \alpha a_{1j} x_{1j}^{\alpha-1} - \lambda_1 = 0, \quad j = 1,2,\dots,n, \tag{15}$$

$$\alpha a_{1j} x_{1j}^{\alpha-1} = \lambda_1, x_{1j} = \left(\frac{\lambda_1}{\alpha a_{1j}}\right)^{\frac{1}{\alpha-1}} =$$

$$= \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \cdot \left(\frac{1}{a_{1j}}\right)^{\frac{1}{\alpha-1}}. \tag{16}$$

Let us find λ_1, by means of using (13)

$$\sum_{j=1}^{n} x_{1j} = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \cdot \sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} = \mathrm{C}_1, \tag{17}$$

whence

$$\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{\mathrm{C}_1}{\sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}, \tag{18}$$

$$x_{1j} = \frac{\mathrm{C}_1 \left(\frac{1}{a_{1j}}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n} \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}} = \frac{\mathrm{C}_1 a_{1j}^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n} (a_{1j})^{\frac{1}{1-\alpha}}},$$

$$j = 1,2,\dots,n. \tag{19}$$

Repeating actions (15)-(18) for x_{2j}, we obtain a similar result:

$$x_{2j} = \frac{\mathrm{C}_2 a_{2j}^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n} (a_{2j})^{\frac{1}{1-\alpha}}}, \quad j = 1,2,\dots,n.$$

Conclusions. A method for solving the problem of rational distribution of a one-dimensional limited resource in the context of fuzzy initial data is proposed and justified. The technology and computational scheme for implementing the method do not depend on the type of objective function of the problem and on the nature of the membership functions of its fuzzy parameters.

References:

1. *Khflilzadeh M.* (2022) Resource leveling in projects considering different activity execution modes and splitting. Journal Engineering. Design and Technology vol.20 No.5, pp. 1073-1100.
2. *Atan T.* (2018) Optimal project duration for resource leveling. European Journal of Operational Research, Elsevier. Vol.266, pp 508-520.
3. *JL. Ponz-Tienda*. The Resource Leveling Problem with multiple resources using on adaptive genebir adorithm. Automation in construction volume 29, 2013, pp 161-172.
4. *T.W. Liao* Metaheuristies for project and construction management. Automation in Construction, 2011, - 64-78.
5. *J. Mandi, P. Stuckey, T. Guns*, et al. Smart predict-and-optimize for hard combinatorial optimization problems. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 34, pp 1603–1610, 2020.
6. *R. Kicsiny* Allocation of limited resources under quadratic constraints. Annals of Operation Research. Vol. 322, pp 793-817, 2023.
7. *M. Yakubov, Y. Gafurov, L. Varlamova* Issues of rational distribution of water resources under deficit conditions. International Scientific Conference “Construction Mechanics, Hydraulics and Water Resources Engineering” (CONMECHYDRO - 2021). E3S Web Conf. Volume 264, 2021.
8. *T. Khatun, K. Hiekata* Dynamic Modeling of Resource Allocation for Project Management in Multi-Project Environment. Transdisciplinary Engineering for Resilience: Responding to System Disruptions. 2021.

Статтю представив д.т.н., проф. Національного технічного університету "Харківський політехнічний інститут" О.А. Серков.

Надійшла (received) 11.03.2024

Raskin Lev, Dr.Sci.Tech.
National Technical University "Kharkiv Polytechnic Institute"
Str. Kyrpychova, 2, Kharkiv, Ukraine, 61000
Tel.: (050) 634-30-60, e-mail: topology@ukr.net
ORCID ID: 0000-0002-9015-4016

Sukhomlyn Larysa, CSc.Tech.
Kremenchuk Mikhail Ostrogradskiy National University
Str. Pershotravneva, 20, Kremenchuk, Ukraine, 39600
Tel.: (050) 956-03-41, e-mail: lar.sukhomlyn@gmail.com
ORCID ID: 0000-0001-9511-5932

Hatunov Artur, master.
National Technical University "Kharkiv Polytechnic Institute"
Str. Kyrpychova, 2, Kharkiv, Ukraine, 61000
Tel.: (098) 251-31-02, e-mail: gatunovartur@gmail.com
ORCID ID: 0009-0006-4444-3817

ISSN 2079-0031 (Print) ISSN 2411-0558 (Online)

Andriienko Serhii, master.
National Technical University "Kharkiv Polytechnic Institute"
Str. Kyrpychova, 2, Kharkiv, Ukraine, 61000
Tel.: (066) 617-77-19, e-mail: sergeyandrienko1206@gmail.com
ORCID ID: 0009-0009-4036-9877

UDC 004.94

Оптимальний розподіл обмеженого ресурсу в умовах невизначеності / Раскін Л., Сухомлин Л., Хатунов А., Андрієнко С. // Вісник НТУ "ХПІ". Серія: Інформатика та моделювання. – Харків: НТУ "ХПІ". – 2024. – № 1 – 2 (11 – 12). – С. 5 – 14.

Об'єкт дослідження: методи раціонального розподілу обмеженого ресурсу в умовах невизначеності. Передбачається, що система складається з набору незалежно функціонуючих елементів, а ефективність цих елементів залежить від рівня інвестицій. Складність вирішення задачі традиційними методами залежить і визначається типом аналітичного опису виробничих функцій елементів системи. Значні труднощі виникають, якщо параметри виробничих функцій є невизначеними величинами, заданими, наприклад, з точки зору нечіткої математики. Ця обставина підкреслює актуальність вивчення підходів до вирішення поставленої задачі розподілу ресурсу для випадку, коли його параметри не є чітко визначеними, а також визначає мету роботи. Проблеми, що випливають із поставленої мети, полягають у розробці математичних моделей і методів раціонального розподілу ресурсів для основних типів виробничих функцій, параметрами яких є нечіткі числа типу (L-R). Для вирішення цих проблем запропоновано метод, який перетворює вихідні нечіткі задачі в зрозумілі, які можна розв’язати за допомогою стандартних технологій оптимізації з обмеженнями. Важливим результатом дослідження є те, що запропонований метод є універсальним, тобто застосовується однаково для вирішення задач розподілу ресурсів у системах, де виробничі функції елементів можуть мати довільний характер. Бібліогр.: 11 назв.

Ключові слова: раціональний розподіл обмеженого ресуру; нечітка математика; нечіткі числа типу ($L – R$).

UDC 004.94

Optimal distribution of limited resource in a context of uncertainty / Raskin L., Sukhomlyn L., Hatunov A., Andriienko S. // Herald of the National Technical University "KhPI". Series of "Informatics and Modeling". – Kharkov: NTU "KhPI". – 2024. – № 1 – 2 (11 – 12). – P. 5 – 14.

Object of research: methods of rational distribution of limited resource in a context of uncertainty. It is assumed that the system consists of a set of independently functioning elements, and effectiveness of these elements depends on the level of investment. The complexity of solving the problem using traditional methods depends on and is determined by the type of analytical description of production functions of the system elements. Significant difficulties arise if parameters of production functions are uncertain quantities, specified, for example, in terms of fuzzy mathematics. This circumstance emphasizes the relevance of studying approaches to solving the set problem of resource distribution for the case when its parameters are not clearly defined, and this circumstance also determines the goal of the work. Problems arising from the goal consist in developing mathematical models and methods for rational resource distribution for the main types of production functions with their parameters being fuzzy numbers of ($L – R$) type. To solve these problems, a method is proposed that transforms the original fuzzy problems into clear ones that can be solved using standard constrained optimization technologies. An important result of the research consists in the fact that the proposed method is universal, that is, it is applied in the same way to solve problems of resource distribution in systems wherein production functions of the elements can be of arbitrary nature. Refs.: 11 titles.

Keywords: rational resource limited distribution; fuzzy parameters; fuzzy mathematics; fuzzy numbers of ($L – R$) type.